The magnetic relaxation effect on TEM responses of a uniform earth

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Abstract

Ungrounded horizontal loop transient responses of uniform conductive and magnetically viscous earth have been simulated using two different codes. One algorithm employs the relationship between viscous magnetization and the magnetic flux it induces in the receiver loop. In the other algorithm, the Helmholtz equation in a boundary-value problem is solved using the Fourier transform with frequency-dependent magnetic permeability. The two solutions are identical for noncoincident loops but differ when the transmitter and receiver loops are closely spaced (at 1 cm or less). In the latter case correct results are provided by the first code. The magnetic relaxation and eddy current responses appear to be independent at conductivities typical of the real subsurface. Therefore, TEM responses of magnetically viscous conductors can be computed using the superposition principle. Although transients change in an intricate way as a function of loop geometry and earth parameters, these changes exhibit certain patterns which may be useful at the stages of exploration and TEM data processing. In configurations where the receiver loop is laid outside the transmitter, the interaction of magnetic relaxation and eddy current decay causes sign reversal in transients. This reversal occurs at late times after an earlier sign reversal due uniquely to eddy current.

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Introduction

Magnetic viscosity is a property of ferromagnetism. In rocks it is associated with superparamagnetism, or magnetic relaxation of ultrafine grains in ferrimagnetic minerals. Magnetic viscosity normally causes a much less effect on TEM data than eddy current. There are, however, natural and man-made objects in which the amount of superparamagnetic particles is as great as to make the magnetic relaxation response notable or even dominant over the conductivity-controlled eddy current response. This is the effect that cannot be ignored in data interpretation.

Magnetic viscosity is most often treated as geologic noise that interferes with TEM responses to be interpreted in terms of “normal” electrical conductivity (Buselli, 1982; Dabas and Skinner, 1993; Lee, 1984a,b; Pasion et al., 2002; Zakharkin et al., 1988; Zakharkin and Bubnov, 1995). On the other hand, there is evidence that magnetic viscosity effects in TEM measurements bear signature of genesis and structure of natural and man-made materials and near-surface processes (Barsukov and Fainberg, 1997, 2002; Kozhevnikov and Nikiforov, 1996, 1999; Kozhevnikov and Snopkov, 1990, 1995; Kozhevnikov et al., 1998, 2001, 2003). Therefore, it appears reasonable to learn how to (i) amplify or damp the magnetic viscosity response, (ii) image lateral and vertical magnetic viscosity profiles in shallow subsurface, and (iii) interpret the results in terms of rock physics and, possibly, magnetic mineralogy.

For this purpose, special tools are required for mathematical modeling of transient responses of magnetically viscous ground, in addition to laboratory and field experiments. The primary objective is to design forward modeling codes to be complemented in the long run with inversion algorithms.

An important contribution to the modeling experience belongs to T.J. Lee who derived analytical equations for transient responses of a conductive superparamagnetic ground (Lee, 1984b) and a thin superparamagnetic layer on top of a conductive nonmagnetic ground (Lee, 1984a). As Lee reported, the magnetic viscosity effect was stronger than in separate loops, especially, in the case where the superparamagnetic material was confined to a thin top layer of the ground (Lee, 1984a). Lee also showed that the coincident-loop transient response depended on the loop area...
as well as on the wire radius, and the wire-radius dependence was more evident in loops on a superparamagnetic ground (Lee, 1984a,b). Lee’s equations (1984a, b) derived for circular loops are critical for understanding the physics of superparamagnetism and are useful to predict the order of the expected effects but they hardly can make the basis for appropriate modeling.

Interest in the magnetic viscosity effect on TEM data has recently been rekindled in applications of UXO (unexploded ordnance) detection (Pasion et al., 2002). The models for UXO detection simulated in-loop transient responses of magnetically viscous materials obtained with a small circular receiver at the center of a relatively small circular transmitter. With small loop systems, the eddy current decay is usually so rapid that it has died out before the first time gate. However, with large square-loop systems in conductive terrain, which is a common case of TEM surveys, the conductivity effect can produce a pronounced response.

As far as we know, there has not been much literature on mathematical modeling of TEM responses of magnetically viscous ground. We failed to find publications that would discuss different models and compare their performance at different resistivity patterns and loop geometries. This modeling, however, will be an indispensable support to TEM soundings of a superparamagnetic ground which can make magnetic viscosity an inversion-derived parameter. We are trying to somehow bridge the gap by this study using the available literature on magnetic viscosity of rocks and our own results, which were partly reported elsewhere in brief communications (Antonov and Kozhevnikov, 2003; Kozhevnikov and Antonov, 2004).

**Magnetic relaxation and its relation with induction transients**

Assume that a transmitter of DC current $I$ has been on indefinitely and the transmitter and receiver loops lie on nonmagnetic ground (Fig. 1). In this case, the magnetic flux $\Phi_0$ induced in the receiver loop is $\Phi_0 = IM_0$, where $M_0$ is the coefficient of inductance between two loops on nonmagnetic half-space.

If there is a magnetic object in the loop vicinity, the primary magnetic field $H_1$ charges its any elementary volume with the magnetization $J$. The magnetized object induces the secondary magnetic field $H_2$ which adds $\Delta \Phi$ to the initial magnetism $\Phi_0$. Correspondingly, $M_0$ changes for the value $\Delta M$ called introduced inductance. It either amplifies or reduces the initial inductance depending on loop geometry and magnetic susceptibility of the ground. Measuring $\Delta M$ can give information on the presence of a magnetic object and on its properties. The inductance that bears the effect of one or several magnetized objects (including magnetic half-space) is called effective inductance ($M_e$). It is convenient to write $M_e$ as

$$M_e = \mu_e M_0,$$  \hspace{1cm} (1)

where $\mu_e$ is the effective relative magnetic permeability which allows for the response of magnetic objects in the loop vicinity and is $\mu_e = 1$ in their absence. There is always such $\mu_e$ that (1) fulfills exactly in the case of a horizontally uniform magnetic earth; otherwise, (1) is approximate.

As the current is turned off instantly at time $t = 0$, the primary magnetic field disappears immediately. Assume that the conductivity of the object and its host is so small that eddy current and the secondary magnetic field it induces decay rapidly on a time scale of the experiment and cause no effect on magnetization, but viscous magnetization decays slowly. Magnetic relaxation excites synchronous secondary magnetic field $H_2$ which induces the voltage $e(t) = -\frac{d\Phi}{dt}$ in the receiver loop.

In the case of single-loop or coincident-loop excitation and measurement, the magnetic flux equation will include the loop inductance $L$ instead of the mutual inductance $M$.

Rock magnetism is often expressed via the magnetic susceptibility $\kappa$ instead of the permeability $\mu$. In the SI system, relative $\mu$ and $\kappa$ are related as

$$\mu = 1 + \kappa.$$  \hspace{1cm} (2)

Correspondingly, their effective counterparts are related as

$$\mu_e = 1 + \kappa_e.$$  \hspace{1cm} (3)

Therefore,

$$M_e = M_0(1 + \kappa_e).$$  \hspace{1cm} (4)

The superparamagnetic decay on removal of the applied magnetic field is slow, and $\mu_e$, $\kappa_e$, and $M_e$ are thus time-dependent. Then, the magnetic flux can be written as Duhamel’s integral:

$$\Phi(t) = I(t)M_e(0) + \int_{-\infty}^{t} I(\tau) \frac{dM_e(t - \tau)}{dt} d\tau.$$
If the transmitter current $I_0$ is turned off at $t = 0$, the time-dependent current $I$ in the loop is $I(t) = I_0[1 - \psi(t)]$, where $\psi(t)$ is the unit Heaviside function. Then,

$$
\Phi(t) = I(t) M(0) + I_0 \int_{-\infty}^{t} \frac{dM_e(t - \tau)}{dt} d\tau - I_0 \int_{-\infty}^{t} 1(t) \frac{dM_e(t - \tau)}{dt} d\tau. \quad (5)
$$

The first integral in the right-hand side of (5)

$$
\int_{-\infty}^{t} \frac{dM_e(t - \tau)}{dt} d\tau = M_e(t) - M_e(-\infty)
$$

obviously gives static effective inductance ($M_{\text{static}}$), i.e., the inductance that sets up in an infinitely long time. The second integral in the right-hand side of (5) is

$$
\int_{-\infty}^{0} 1(t) \frac{dM_e(t - \tau)}{dt} d\tau = \int_{0}^{t} \frac{dM_e(t - \tau)}{dt} d\tau = M_e(t) - M_e(0).
$$

Inasmuch as viscous magnetization is zero at $t = 0$, $M_e(0) = M_0$, i.e., effective inductance equals initial inductance. Thus, (5) becomes

$$
\Phi(t) = I_0 M_s - I(t) I_0 M_0 - I_0 M_e(t),
$$

while the voltage at the loop outputs is

$$
e(t) = -\frac{d\Phi}{dt} = \delta(t) I_0 M_0 + I_0 \frac{dM_e(t)}{dt},
$$

where $\delta(t)$ is the Dirac delta function. $M_e(t)$ can be expressed via the effective time-dependent magnetic susceptibility $\kappa_e(t)$ and the initial inductance $M_0$ as

$$
M_e(t) = M_0[1 + \kappa_e(t)].
$$

Therefore,

$$
e(t) = -\frac{d\Phi}{dt} = \delta(t) I_0 M_0 + I_0 M_0 \frac{d\kappa_e}{dt}.
$$

There the first term is the voltage induced in the receiver loop on the transmitter turn-off which bears no information on magnetic viscosity. The current-normalized voltage induced in the receiver by magnetic relaxation is

$$
\frac{e(t)}{I_0} + M_0 \frac{d\kappa_e}{dt}. \quad (6)
$$

To use (6) in practice, one has to calculate $M_0$ and specify the model for $\kappa_e(t)$.

Magnetic susceptibility of an ensemble of single-domain particles

An external magnetic field applied to a “normal” material magnetizes it immediately, i.e., the applied field $H$ and the magnetization $J$ are in phase and are related as $J = \kappa H$, where $\kappa$ is time independent.

Magnetization of superparamagnetic materials is time-dependent. For a magnetic field applied at $t = 0$, $J(t) = \kappa(t) H$, where $\kappa(t)$ is time-dependent magnetic susceptibility. $J(t)$ is often written as

$$
J(t) = \kappa_0 H [1 - P(t)], \quad (7)
$$

where $\kappa_0$ is the static susceptibility and $P(t)$ is the after-effect function (Trukhin, 1973).

Magnetization of a single-domain grain has the relaxation time $\tau = \tau_0 \exp(J H / k T)$, where $K$ is the anisotropy energy, $V$ is the particle volume, $T$ is the absolute temperature, $k$ is Boltzmann’s constant, and, $\tau_0 = 10^{-9}$ s (Neel, 1949). For a particle or an ensemble of particles with the same time constant, the after-effect function is $P(t) = \exp(-t/\tau)$ (Trukhin, 1973).

Relaxation times associated with superparamagnetic behavior of minerals in nature are in a range defined by the weight function $f(\tau)$ also called the distribution function. Then, the after-effect function is

$$
P(t) = \int_{0}^{\infty} f(\tau) \exp(-t/\tau) d\tau. \quad (8)
$$

The distribution of time constants in an ensemble of single-domain particles with uniformly distributed energy barriers between stable magnetization states is described by the Fröhlich function (Fannin and Charles, 1995). The relaxation times $\tau$ in this function are in the range from $\tau_1$ to $\tau_2$: $\tau_1 \leq \tau \leq \tau_2$. Inside the range,

$$
f(\tau) = \frac{1}{\tau \ln(\tau_2/\tau_1)}
$$

and outside it $f(\tau) = 0$.

If the argument of (9) is $\ln \tau$, the Fröhlich function becomes

$$
G(\ln \tau) = \frac{1}{\ln(\tau_2/\tau_1)},
$$

whence the relaxation times are uniformly distributed between $\tau_1$ and $\tau_2$.

Substituting (9) into (8) gives

$$
P(t) = \frac{1}{\ln(\tau_2/\tau_1)} \int_{\tau_1}^{\tau_2} \exp(-\tau/\tau) \frac{d\tau}{\tau}.
$$

(10)

The exact values of $\tau_1$ and $\tau_2$ are commonly unknown but this is of no significance in actual measurements. The range of time constants normally covers many orders of magnitude while magnetic relaxation is measured at $t_1 \ll t \ll t_2$. Then, the after-effect function is (Fannin and Charles, 1995)

$$
P(t) = \frac{1}{\ln(\tau_2/\tau_1)} (-\gamma - \ln t - \ln \tau_2),
$$

where $\gamma = 0.577$ is the Euler constant. Substituting (10) into (7) gives that the magnetization of a ground excited by a step external field increases proportionally to the logarithm of time:

$$
J(t) = \frac{\kappa_0 H}{\ln(\tau_2/\tau_1)} [1 + A + \ln t],
$$

where $A = \gamma + \ln \tau_2$. Dividing both sides of the equation by $H$ gives the time-dependent susceptibility

$$
\kappa(t) = \frac{J(t)}{H}.
$$
\[ \kappa(t) = \frac{\kappa_0}{\ln (\tau_e/\tau_i)} (1 - A + \ln t). \]  
\[ \text{(11)} \]

Note. The time-dependent susceptibility is meant in this study as a response to magnetic field turn-on and can be thus denoted \( \kappa_{on}(t) \). This is an increasing time function. In some publications, it may correspond to a turn-off response and denoted \( \kappa_{off}(t) \). Obviously, \( \kappa_{off}(t) = \kappa_0 - \kappa_{on}(t) \).

Susceptibility of superparamagnetic materials was shown to be complex and frequency-dependent (Worm, 1999). In the frequency domain, the susceptibility of an ensemble of single-domain grains with their time constants satisfying (9) is given by (Fanning and Charles, 1995; Lee, 1984a,b; Trukhin, 1973)

\[ \kappa^2(\omega) = \kappa_0 \left[ 1 - \frac{1}{\ln (\tau_e/\tau_i)} \ln \left( \frac{1 + j\omega \tau_e}{1 + j\omega \tau_i} \right) \right], \]
\[ \text{(12)} \]

where \( j = \sqrt{-1} \), \( \omega \) is the angular frequency.

The susceptibility of the form (12) approaches the static \( \kappa_0 \) at low frequencies and tends to zero at high frequencies. At frequencies \( 1/\tau_2 < \omega < 1/\tau_1 \), the real component \( \kappa^2(\omega) \) decreases proportionally to the logarithm of frequency and the imaginary component is frequency-independent (Fanning and Charles, 1995).  

**Uniform earth: effective permeability and transient response**

Below we derive the equation for the transient response of a uniform conductive earth that also exhibits magnetic viscosity.

The inductance between two loops laid on a uniform ground with the relative permeability \( \mu \), as well as the inductance of each loop, increase by a factor of \( 2\mu/(\mu + 1) \), i.e., the effective (\( \mu_e \)) and true (\( \mu \)) relative permeabilities of a uniform half-space are related as (Spies and Frischknecht, 1991)

\[ \mu_e = \frac{2\mu}{\mu + 1}. \]
\[ \text{(13)} \]

Taking into account (2), (3) and (13), it is easy to show that the effective susceptibility \( \kappa_e \) of a uniform half-space is related to its true susceptibility \( \kappa \) as \( \kappa_e = \kappa/\kappa + 2 \).

For most geological conditions, \( \kappa \ll 1 \), and \( \kappa_e = \kappa/2 \).

The effective permeability of a ground with time-dependent susceptibility is likewise time-dependent, i.e., \( \kappa_e(t) = \kappa(t)/2 \), and, according to (6),

\[ \frac{\epsilon(t)}{M_0} = \frac{1}{2} \frac{\kappa_0}{\ln (\tau_e/\tau_i)} \frac{1}{t}. \]
\[ \text{(14)} \]

In order to use (14) for estimating the transient response of a uniform magnetically viscous ground, one has to calculate \( M_0 \) (\( L_0 \) for single-loop or coincident-loop configurations) and to set the static susceptibility \( \kappa_0 \) and the \( \tau_2/\tau_1 \) ratio.

Inversion becomes possible when measured transients are available. In (14) \( \epsilon(t) \) is a measured value and \( M_0, L_0 \), and \( t \) are the derived values. Solving (13) with respect to \( \kappa_0 \) gives

\[ \kappa_0 = \frac{2}{L_0} \frac{1}{\kappa_0} \frac{1}{\ln (\tau_e/\tau_i)} \frac{1}{t}. \]
\[ \text{(15)} \]

Thus, measuring \( \epsilon(t) \) can give \( \kappa_0 \). The susceptibility \( \kappa_0 \) equals the true static susceptibility in a uniform half-space and is an effective parameter in a nonuniform half-space, where it can be reasonably called apparent static susceptibility.

Note. The susceptibility \( \kappa_0 \) of a uniform half-space and the apparent static susceptibility of a nonuniform half-space are controlled by the logarithmic \( \tau_2/\tau_1 \) ratio in (14) and (15). Therefore, when reporting the results, one has to specify which \( \tau_2 \) and \( \tau_1 \) have been used in forward modeling or in inversion.

**Algorithm based on the Fourier boundary-value solution**

We consider a layered earth with the conductivities \( \sigma_1, ..., \sigma_r, ..., \sigma_N \), permeabilities \( \mu_1, ..., \mu_r, ..., \mu_N \), and the layer interfaces at \( z_1, ..., z_r, ..., z_N \), in the Cartesian coordinates \( xyz \), where \( z = \text{downward} \). The field of an arbitrary point source in a layered subsurface can be found from the spatial Fourier images of its vertical components (Tabarovsky, 1975).

A model loop system consists of horizontal electric dipoles placed along the contour of the transmitter loop. The voltage \( \epsilon(t) \) induced in the receiver loop \( L_r = L_0(r) \) on the removal of the transmitter field \( L_t = L_0(r_0) \) is found by double integration:

\[ \epsilon = \int L_t \int E(I/dI_0, |r_0 - r|) dr_0 dr, \]
\[ L_r L_t \]

where \( I \) is the current, \( E(I/dI_0, |r_0 - r|) \) is the field of the electric dipole with the moment \( I/dI_0 \) located at the point \( r_0 = (x_0, y_0, z_0) \) of the transmitter loop \( L_t \) and measured at \( r = (x, y, z) \) of the receiver loop \( L_r \). The transients generated by an inductive loop system can be calculated with regard to only the magnetic mode of the electrical point sources. The corresponding frequency-domain components of the electric field are given by

\[ E_x = \frac{i\omega M_0}{4\pi} \frac{\partial^2}{\partial z^2} \int f(u, \omega, z, z_0) J_0(u|lr - r|) u du, \]
\[ E_y = \frac{i\omega M_0}{4\pi} \frac{\partial^2}{\partial z^2} \int f(u, \omega, z, z_0) J_0(u|lr - r|) u du, \]
\[ E_z = \frac{i\omega M_0}{4\pi} \frac{\partial^2}{\partial z^2} \int f(u, \omega, z, z_0) J_0(u|lr - r|) u du. \]
\[ \text{(16)} \]
\[ \text{(17)} \]
\[ \text{(18)} \]

There \( u = \sqrt{k_x^2 + k_y^2} \), \( k_x, k_y \), where \( k_x \) and \( k_y \) are the Fourier images of the spatial frequencies along the horizontal coordinates and \( J_0 \) is the zero-order Bessel function of the first kind.
The components of the randomly oriented source are expressed using a set of trigonometric functions. The integrand function $f(u, \omega, z, \zeta_0)$ governs the field dependence on the earth parameters, including $\sigma, \mu$, the interfaces $(z_1, z_2, \ldots, z_{N-1})$, the source and receiver depths $\zeta_0, z$, and the frequency $\omega$. The loop geometry is included in the Bessel function argument.

The function $f(u, \omega, z, \zeta_0)$ is found using the recurrent formulas (Tabarovsky, 1979):

$$\alpha_N = 0,$$

$$\beta_j = \begin{cases} \frac{\mu_j \rho_{j-1} \alpha_j + 1}{\mu_{j-1} \rho_j \alpha_j - 1}, & j \geq 1, \\ 1, & j = 0 \end{cases},$$

$$R_j = \begin{cases} 1 + \beta_j & j \geq 1, \\ 1 - \beta_j, & j = 0 \end{cases},$$

$$\alpha_{j-1} = -e^{2p_0(j-1)(z_{j-1} - z_{j-2})} R_j^{-1},$$

$$\beta_{j-1} = \frac{e^{2p_0(j-1)(z_{j-1} - z_{j-2})}}{2p_0} R_j^{-1}.$$

Integrals (16)–(18) are calculated using special spline interpolation quadratures, and the quadrature coefficients for point sources are found once and saved in a special file. For an arbitrarily configured system, it is enough to integrate the quadrature coefficients for a point source. This approach speeds up computing the TEM responses for arbitrary loop geometries.

Software summary

The Unv_QQ program designed by E.Yu. Antonov in FORTRAN implements the above algorithm. It is applicable to compute the transient responses of magnetically viscous layered conductors for any rectangular-loop system. In addition to loop geometry and layer parameters, the user can specify the loop height above the earth surface. The magnetic viscosity effects are included using the frequency-dependent complex permeability $\mu_e(\omega) = \mu_0[1 + \kappa_e(\omega)]$, where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the vacuum permeability, and $\kappa_e(\omega)$ is the susceptibility defined by (12).

The MVIS program designed by N.O. Kozevnikov in the MATHCAD environment is used to compute transients with magnetic relaxation using equation (6) for the following loop configuration and earth models:

- loops of any geometry, uniform earth;
- circular coincident loops, two-layer earth;
- circular or square noncoincident in-loop system, layered earth with any number of layers.

Equation (6) was derived assuming a low-conductive subsurface in which eddy current decays very rapidly and, hence, causes no effect on the magnetic relaxation response. This may seem to be a limitation for MVIS making it restricted to high-resistivity, actually zero-conductivity, terrains. However, we show below that the eddy current and magnetic relaxation responses are independent in the conductivity range of real earth.

Time-dependent inductance and numerical method: comparing two approaches

At the first stage of the reported numerical experiments we compared the transients computed by the Unv_QQ and MVIS codes for the same loop systems and earth models. The results turned out to be identical for in-loop but different for coincident-loop transients.

To understand the cause of this difference, we computed in-loop transients for a system with a receiver loop of a side length varying from 90 to 100 m placed inside a 100-m square transmitter loop, i.e., a noncoincident-loop configuration graded into a coincident one.

We used a uniform earth model with a resistivity $\rho = 10^6$ Ohm-m and $\kappa_0 = 10^{-3}$ SI units. At this resistivity the effect of fast-decaying eddy current is vanishing relative to the magnetic relaxation response. It was assumed, both in calculation (14) and in numerical experiments, that $\tau_1 = 1 \times 10^{-6}$ s, $\tau_2 = 1 \times 10^6$ s, and delay times between 10 ms and 100 ms, i.e., $\tau_1 < \tau < \tau_2$.

The inductance $M_0$ between the square loops was found using numerical integration of (3.12) from (Nemtsova, 1989). The loop inductance $L_0$ for coincident-loop transients was obtained with the equation from (Zimin and Kochanov, 1985, p. 163).

See the $e_1/e_2$ curves computed using MVIS and Unv_QQ in Fig. 2, where $e_1$ and $e_2$, respectively, are current-normalized. The two solutions diverged notably when the receiver loop became in close proximity of the transmitter (at about 1 cm). As they further approached one another, the $e_1/e_2$ ratio increased to become 1.9 in the limit, when the two loops coincided. The $e_1/e_2$ ratio turned out to be independent of time delay within the range from 10 ms to 100 ms.

Inasmuch as the two solutions were identical at loop spacing over 1 cm, we reasonably supposed that both approaches drove at the correct result, taking into account that they based on different methods.

What happens when the loops become closely spaced? See Fig. 2, b for the $M_0$ behavior of two square loops centered on the same point. The size of the outer loop remains invariable (1000100 m) while the inner loop grows from 1 x 10 m to 100 x 100 m, and $M_0$ increases proportionally to the loop area. As the loops approach one another, $M_0$ grows ever more rapidly and reaches its maximum when they coincide. Then, $M_0$ becomes $L_0$ of the 100-m loop which, unlike $M_0$, depends on both the loop size and the wire thickness. At a wire radius of 2 mm, $L_0 = 8.24 \times 10^{-4}$ H. Then, $M_0$ reaches half the $L_0$ when the receiver loop is 99.5 x 99.5 m. To put it different, inductance between the two loops becomes half the maximum value when they are spaced at only 25 cm.
According to (14), the magnetic relaxation response is proportional to the initial inductance $M_0$ of the transmitter and receiver loops. $M_0$ grows rapidly when the loops approach to a distance of a few centimeters and less (Fig. 2, b) and is the greatest in coincident loops. Therefore, coincident-loop systems are the most sensitive to magnetic viscosity.

The voltage induced in the receiver loop by eddy current is proportional to inductance between the receiver and the ground eddy current which can never coincide with the transmitter loop in any conditions. Eddy current diffusion can be illustrated by an equivalent current smoke-ring that grows in size and in depth (Nabighian, 1979). There is inductance between any two elementary ring filaments controlled by their size and position and by the ground magnetic properties. This interaction and, hence, the ground magnetic properties, are taken into account in the Unv_QQ solution. However, the eddy current maximum is far from the loop wire already at the earliest times. Therefore, the Unv_QQ solution does not include magnetization of ground in the immediate loop vicinity.

**Results and discussion**

Having explored the potentialities and the limitations of the two approaches, we discuss some results that may be useful in exploration and in processing the TEM responses of a magnetically viscous ground.

Figure 3 shows noncoincident-loop transient responses of a uniform earth measured with a 50-m square receiver inside a 100-m transmitter. First we computed the eddy current induced voltage $e_1(t)/I$ assuming $\rho = 10$, $10^2$, and $10^3$ Ohm-m, $\kappa_0 = 0$ (no magnetic viscosity).

Then we found the magnetic relaxation response $e_2(t)/I$ assuming $\kappa_0 = 0.001$ and the resistivity $\rho = 10^5$ Ohm-m at which eddy current decays so fast that the transient is controlled uniquely by magnetic viscosity.

Finally, we obtained the total response of eddy current plus magnetic relaxation. See the $e_2(t)/I = e_1(t)/I + e_2(t)/I$ curves in Fig. 3, together with the $e(t)/I$ curves computed using Unv_QQ with regard to eddy current–magnetic relaxation interaction, at $\kappa_0 = 0.001$, $\rho = 10$, $10^2$, and $10^3$ Ohm-m. The $e_2(t)/I$ and $e(t)/I$ curves coincide, which proves the independence of the magnetic relaxation and eddy current responses (Kozhevnikov and Snopkov, 1990, 1995). Therefore, the superposition principle is applicable to computing and interpreting transients of magnetically viscous conductors.

The resistivity $\rho$ in early-time transients controls the eddy current response (Fig. 3) while at late times the curves follow the asymptote corresponding to the magnetic relaxation response. The higher the resistivity the earlier the time when the curves turn to and reach the asymptote. The transient process in nonconductive earth is obviously controlled uniquely by its magnetic viscosity. The decay of magnetization is $1/t$, i.e., is much slower than in the eddy current response. The magnetic viscosity effect shows up in the behavior of apparent resistivity ($\rho_\tau$) derived from the transients (Fig. 4) as a continuous $\rho_\tau$ fall-off to the $1/t$ asymptote. The higher the resistivity, the earlier the fall-off becomes notable.
It is essential to figure out how the magnetic viscosity effect depends on the loop geometry and size in TEM soundings of a ground that may be superparamagnetic. Figure 5 shows in-loop transient responses of a uniform earth with ρ = 10^3 Ohm-m, κ₀ = 10⁻³ SI units, τ₁ = 10⁻⁶ s, τ₂ = 10⁶ s. The transmitter loop side varied from 10 to 10³ m and the receiver was always 10×10 m.

As the loop size increased, the eddy current effect increased as well but the magnetic viscosity effect decreased (Fig. 5, a). Mind that the magnetic relaxation response is proportional to the transmitter-receiver inductance. At an invariable receiver size, this induction is the greatest when the loops coincide (see Fig. 5, b for the corresponding apparent resistivity curves). Therefore, the magnetic viscosity effect can be highlighted by using small coincident loops and damped, if unwanted, in noncoincident loop transients, with a small receiver inside a large transmitter.

If we assume that the inductance is positive (M₀ > 0) for a receiver inside a transmitter, it will be negative (M₀ < 0) if the receiver is placed outside the transmitter (Nemtsov, 1989). The magnetic viscosity-controlled transient has the same sign as a “normal” transient in the former case and there is a sign reversal in the latter case (Fig. 6, a). The system of Fig. 6, a consists of a 50-m square receiver loop placed outside a 100-m square transmitter loop, at a distance of 80 m between the loop centers; the loops lie on a uniform ground with ρ = 10⁵ Ohm-m and κ₀ = 10⁻³ SI units. See a sign reversal at about 1 ms caused by interaction of eddy current decay and magnetic relaxation (Fig. 7). When the transmitter is on, it produces a
primary magnetic field that causes the magnetization $J$ in rocks. In an isotropic earth, at $\kappa_0 < 1$, the direction of this magnetization follows that of the primary transmitted field at any point of the subsurface.

The removal of the transmitter field launches two processes: magnetic relaxation and eddy current decay. Magnetic relaxation is simultaneous at all points of the subsurface, i.e., the magnetization ratio of any two points remains invariable. This ratio is defined by the transmitter field and earth’s $\kappa_0(x, y, z)$ patterns. See that the secondary magnetic field always aligns with the primary field everywhere at the ground surface (Fig. 7).

An eddy current response illustrated by an equivalent current smoke-ring (Nabighian, 1979) is as follows. First it occurs under the transmitter loop and is approximately of the same size, and the magnetic field it induces inside (point 1) and outside (point 2) the transmitter aligns with the primary field. With the time on, the ring diffuses laterally and depthward, and once it grows to the size when the surface projection of the current line falls at point 2, the magnetic field induced by eddy current changes its polarity. (In our case, the sign reversal occurs at less than 10 $\mu$s and is beyond the transient of Fig. 6.)

The field eddy current induces around point 2 is greater than the superparamagnetically induced field of opposite direction. However, the field induced by the current ring at point 2 decreases as the eddy current diffuses down and laterally decaying by heat loss. Inasmuch as magnetic relaxation is much slower than the decay of eddy current, its field will exceed that of eddy current since some moment of time. That is the point of sign reversal of the total field which keeps decaying to finally reach its minimum; then it decays to zero remaining negative. The transient experiences sign reversal at the point when the total field is minimum and then remains negative decaying further as $1/t$ (Fig. 6, a).

Above we mentioned that in noncoincident loop configurations $M_0 > 0$ when the receiver is inside the transmitter and $M_0 < 0$ when it is laid outside. Thus, there always must be a receiver position corresponding to $M_0 = 0$ (Nemtsov, 1989), i.e., the magnetic viscosity effect becomes eliminated in a magnetically uniform earth. Inasmuch as TEM measurements...
are most often taken at late times, the displacement of the transmitter loop causes no influence on the eddy current response.

Conclusions

Due regard for magnetic viscosity of the ground is a topical problem in TEM surveys. We modeled the magnetic relaxation effect on transient responses of uniform earth with two algorithms. One code employed the relationship between viscous magnetization and the magnetic flux it induces in the receiver loop. This is a simple solution, easy to illustrate physically, but it is not rigorous as it neglects interaction between eddy current and magnetic relaxation.

The other algorithm was based on the numerical solution to the Helmholtz equation taking into account the eddy current–magnetic viscosity interaction. Transient responses computed with the two codes for the same loop configurations and earth models were identical if the transmitter and receiver loops were spaced at more than a few centimeters but differed when the spacing reduced to 1 cm and less. Therefore, both methods provided correct results for noncoincident-loop configurations while the former algorithm was more workable in the case of coincident loops. The magnetic relaxation and eddy current responses were shown to be independent at conductivities common to the real subsurface. Therefore, TEM responses of magnetically viscous conductors can be computed using the superposition principle. Transient responses of a magnetically viscous conductive earth changed in an intricate way as a function of loop geometry and earth parameters but the changes exhibited certain patterns which may be useful at the stages of exploration and TEM data processing.

In noncoincident loop configurations, with a receiver outside the transmitter, the interaction of magnetic relaxation and eddy current decay caused sign reversal in transients. The reversal occurred after an earlier sign reversal due uniquely to the eddy current decay.

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